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Neutrino masses in 5D orbifold SU(5) unification models without right-handed singlets

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Abstract: We explore a mechanism for radiatively generating neutrino Majorana masses in a 5 dimensional orbifold SU(5) unification model without introducing right-handed singlets. The model is non-supersymmetric and the extra dimension is compactified via a $S_1/(Z_2 \times Z_2')$ orbifold geometry. The necessary lepton number violating interaction arises from the Yukawa interactions either between a 10-plet or a 15-plet bulk scalar field and the fermion quintuplets which are residents on the SU(5) symmetrical brane located at one of the orbifold fixed points. The model is engineered to give realistic charged fermion masses and mixing and in the same time avoiding the rapid proton and neutron decays by geometric construction. The gauge unification can be maintained by adding extra fermion or scalar fields. The unification scale is found to be larger then 10^{15} GeV by adding a bulk vector decuplet pair whose zero mode has masses around $10 \sim 100$ TeV range. We found that neutrino mass matrix of the normal hierarchy type is favored by using 15-plet scalar. We give a solution of this type which has detectable $\mu \to 3e$ transition. On the other hand, by introducing 10-plet scalar, the leading neutrino mass matrix can only be inverted hierarchical and gives at most bi-maximal mixing.

Keywords: Field Theories in Higher Dimensions, GUT, Neutrino Physics.

Contents

1. Introduction

Recent measurements of atmospheric and solar neutrino fluxes at the Super Kamiokande [1] and the Sudbury Neutrino Observatory [2] have provided compelling evidence for neutrino masses and neutrino oscillations. This received further support from the reactor KamLand experiment [3]. Furthermore, the Wilkinson Microwave Anisotropy Probe [4] imposes the constraint that the sum of neutrino masses to be less than .75 eV. Such a small value for neutrino masses is generally considered to be a harbinger of new physics beyond the standard model (SM) and the existence of a new scale between the Fermi and the Planck scale. In particular if the three neutrinos involved in weak interactions are Majorana in nature then clearly new physics is at play. The most popular suggestion of generating neutrino masses in the milli-electronvolt range is grand unified theories (GUTs) via the seesaw mechanism with or without supersymmetry. Central to this idea is the introduction of one right-handed singlet neutrino per family of the SM fermions with a mass near the GUT scale. This is natural in SO(10) models since its fundamental **16** representation encompasses this singlet with the 15 fermions of the SM. For a recent review of neutrino masses in grand unified models see [5]. On the other hand small Dirac neutrino masses is considered unnatural due to the extreme fine tuning required. However, in theories with extra dimensions this can be generated by allowing the singlet neutrinos to be bulk fields. A small Yukawa coupling can be obtained due to the volume dilution factor if the extra dimensions are sufficiently large [6]. In both cases right-handed singlet fields N_R are necessary.

In this paper we study the construction of neutrino mass without the benefit of N_R in the context of grand unified SU(5) models with the minimal particle content.[1]. This is

[1]It is also possible to generate neutrino masses without using N_R by R-parity violating interaction in minimal supersymmetric standard model [7]. Here we concentrate on non-supersymmetric models and leave questions such as quantum stability of scalar fields and naturalness issues as unsolved.

a fundamentally different mechanism from the above mentioned constructions. Since the neutrinos in this scenario are Weyl particles of the SM and the resulting neutrino mass matrix is necessarily Majorana. In conventional four dimensional (4D) field theories one can use SM Higgs triplet to achieve this. However, the vacuum expectation value (VEV) of the triplet must be fine tuned to small values in order not to upset the highly successful custodial SU(2) relation of the SM gauge bosons as well as to generate of a sufficiently small neutrino mass. A second method is to radiatively generate neutrino masses at 1-loop. The prototype model was constructed some time ago [8] and the crucial ingredient is the introduction of a SU(2) singlet scalar field with non trivial weak hypercharge. The original version of the model gives a 3×3 neutrino mass matrix with zero diagonal elements and thus leading to bi-maximal neutrino mixings [9]. This is ruled out by the data. However, simple phenomenological modifications can bring it to agree with observations [10]. All these constructions suffer from being rather ad hoc. It will be interesting if one can incorporate these attempts into the theoretically well motivated unification models. A more modern formulation of this makes use of progress in recent works on extra dimensions and the brane world scenario. It has been shown that the technique of orbifold projections applied to GUTs [11] can solve some of the long standing problems such as doublet-triplet splitting that plagued 4D SU(5). This is further applied to the flavor problem by various workers [12]. However, in these works neutrino mass matrices are constructed using right-handed singlets along the line of the seesaw mechanism.

In a previous paper [13] we constructed a viable model of radiative neutrino masses with minimal SM matter in a five dimensional (5D) field theory on the orbifold $S_1/(Z_2 \times Z_2')$ with bulk $\mathrm{SU}(3)_W$ gauge symmetry. The $\mathrm{SU}(3)_W$ unifies the $\mathrm{SU}(2) \times \mathrm{U}(1)$ electroweak symmetry of the SM [14]. The crucial observation is that the SM lepton doublet and charged lepton singlet can naturally be embedded in the fundamental representation of $\mathrm{SU}(3)_W$. With Higgs fields in the **6** and **3** representations lepton number violating interactions can be constructed. Tree level masses are forbidden by orbifold projections and thus avoiding the fining tuning of VEV for small neutrino masses. Neutrino masses can be generated at 1-loop level. The scale of neutrino masses is small compared to the weak scale due to three factors: (i) the loop factor, (ii) the inverse of the compactification radius, R, which controls the volume dilution factor and (iii) small Yukawa couplings of the charged leptons. We found neutrino masses of order 0.01 eV without much fine tuning. Interestingly the solutions that satisfy the observed mixing parameters are found for the inverted mass hierarchy type if Yukawa couplings are not fine tuned.

We continue this investigation for the GUT theory of SU(5) which is theoretically well motivated. This poses new challenges since the fundamental representation **5** unifies the lepton doublet $(\nu\ e)_L$ with the right-handed down type quark d_R. We found two options for the Higgs fields that can give rise to lepton number violating interactions essential for our mechanism. Strictly speaking the concept of lepton number is not fundamental in these theories; however, we find it convenient to use it both for guiding model construction as well as in navigating the tight constraints imposed by the many rare decay experiments. The Higgs fields are the **15** or **10** representation. Another problem we encountered in this construction is to maintain gauge coupling unification since it is well known that exotic

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Higgs contributes to the running of the coupling constants. This solved by introducing bulk fermion fields or additional Higgs fields.

This paper is organized as follows: In section 2, we will review the setup of 5D orbifold SU(5) theory. The details of the model are given here. The construction of the Majorana neutrino masses through the Yukawa interaction of bulk **10** or **15** Higgs is made explicit in section 3. The gauge unification question will be addressed in section 4. The rich exotic Higgs sector leads to interesting phenomenology and will be discussed in section 5. Finally we give our conclusions in section 6.

2. 5D orbifold SU(5) model

We begin by a brief review of the 5D SU(5) unification model defined on the orbifold $S_1/(Z_2 \times Z_2')$ [15] with coordinates $x^\mu (\mu = 0, 1, 2, 3)$ and the extra spatial dimension which is denoted by y. The circle S_1 of radius R, or $y = [-\pi R, \pi R]$, is orbifolded by parity $P : y \leftrightarrow -y$ transformation. The resulting space is divided by a second Z_2' acting on $y' = y - \pi R/2$ as $P' : y' \leftrightarrow -y'$ to give the final geometry. On this orbifold, the Fourier decomposition is summarized in table 1.

Under the $Z_2 \times Z_2'$ transformation, there are two fixed points at $y = 0$ and $y = \pi R/2$ denoted by y_S and y_G respectively. The following two parities matrices are chosen for the parity transformations:

$$\begin{aligned} P &= \mathrm{diag}\{+++++\}\,, \\ P' &= \mathrm{diag}\{---++\}\,. \end{aligned} \tag{2.1}$$

These determine the Kaluza-Klein decompositions of a generic bulk field $A(x, y)$ (see table 1). The parity assignments of a bulk field are chosen by phenomenological considerations. Indeed for a given multiplet different components can have different parities under $Z_2 \times Z_2'$. In particular, for the SU(5) gauge fields $A_M (M = \mu, y)$, the following parities are used

$$\begin{aligned} Z_2 :&\quad A_\mu \to +PA_\mu P^{-1}\,, \qquad A_y \to -PA_y P^{-1} \\ Z_2' :&\quad A_\mu \to +P'A_\mu P'^{-1}\,, \qquad A_y \to \pm P'A_y P'^{-1}\,. \end{aligned} \tag{2.2}$$

where we have suppressed the group index. Since the fifth components vanish at the y_S fixed point they will not enter the low energy effective theory. With this parity assignment,

(P, P')	form	mass
$(++)$	$\frac{1}{\sqrt{2\pi R}}[A_0(x) + \sqrt{2}\sum_{n=1} A_{2n}^{++}(x)\cos\frac{2ny}{R}]$	$2n/R$
$(+-)$	$\frac{1}{\sqrt{2\pi R}}[\sqrt{2}\sum_{n=1} A_{2n-1}^{+-}(x)\cos\frac{(2n-1)y}{R}]$	$(2n-1)/R$
$(-+)$	$\frac{1}{\sqrt{2\pi R}}[\sqrt{2}\sum_{n=1} A_{2n-1}^{-+}(x)\sin\frac{(2n-1)y}{R}]$	$(2n-1)/R$
$(--)$	$\frac{1}{\sqrt{2\pi R}}[\sqrt{2}\sum_{n=1} A_{2n}^{--}(x)\sin\frac{2ny}{R}]$	$2n/R$

Table 1: KK decomposition of a bulk field $A(x, y)$ with parities (P, P').

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when decomposed into the SM subgroup $G = \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, the SU(5) gauge bosons have following components:

$$24 = (8,1,0)_{++} + (1,3,0)_{++} + (1,1,0)_{++} + \left(3,2,-\frac{5}{6}\right)_{+-} + \left(\bar{3},2,\frac{5}{6}\right)_{+-} \tag{2.3}$$

where hypercharge is normalized to $Y = Q - T_3$. The parities (P, P') are shown as subscripts. The first three terms are the SM gauge bosons. With the assigned $(++)$ parities they will have zero modes. On the other hand, the last two terms represent the gauge boson Xs and Ys which have no zero modes since their parities are $(+-)$. They are KK excitations. At the symmetry broken brane, y_G, even these Kaluza-Klein(KK) excitations vanish. So at y_G, the symmetry is always G. On the other hand, at the symmetric brane, y_S, SU(5) is still a good symmetry. At the low energy, only the zero modes of G can be observed. When energy is higher then $1/R$, the appearance of KK excitations of $\mathrm{SU}(5)/G$ plus the KK excitations of G will restore the full SU(5) symmetry. By geometrical construction, the gauge X, Y gauge bosons are massive since they are KK excitations and the lightest ones have masses $\sim 1/R$. At y_G the bulk SU(5) symmetry is broken by the orbifold boundary conditions. So there is no need to introduce the **24** as in the 4D case. Thus, the triplet-doublet problem [16] is solved naturally [11]. To break G to $\mathrm{SU}(3)\times\mathrm{U}(1)$ and to generate fermions masses, one still needs one Higgs in SU(5) fundamental representation.

The SM fermions can be grouped into the standard $\bar{\mathbf{5}}$ and **10** (henceforth denoted as F and T respectively) representations: $F_i = (D_i^c, L_i)$ and $T_i = (U_i^c, Q_i, E_i^c)$ where we use left-handed chiral fields with obvious notations and $i = 1, 2, 3$ is the family index whereas the superscript c denotes charge conjugation. In a 5D theory they can either be bulk fields or brane fields. If the matter fields are placed at the symmetric brane they will enjoy the merits and suffer the drawbacks of SU(5) GUT: the quantization of hypercharge, the unification of gauge couplings, the prediction of mass ratio of down-type quark and charged lepton, and the baryon number violating decays [17]. In a 5D model the possibility that some fermions can be bulk fields and plus the existence of different fixed points to locate fermions can be used to retain the successful mass relationships and avoid some difficulties such as the proton decay problem. For simplicity we shall assume that the F_i are localized on a brane at y_S. Hence, there will be no KK excitations of neutrinos.

Because the prediction of m_b/m_τ agrees quite well with experiment it is reasonable to localize both T_3 and F_3 on the brane at y_S. On the other hand, we can make T_1 a bulk field so as to avoid rapid proton decay, see figure 1. This is so because both T_1 and the X, Y bosons are bulk fields and their interaction must honor the conservation of KK number; i.e. the X, Y bosons are KK modes they do not couple to two up-quark zero modes. Therefore, all the sub-diagrams vanish in the 5D model. Similar diagrams which lead to baryon number violating neutron decays are also forbidden by the KK number conservation and hence matter stability is not violated.

As pointed out in [18] the parities of bulk fermion fields pose further complications. If T_1 has parities $(++)$, then the parities of its components will be $(U_1^c(++)$, $Q_1(+-)$, $E_1^c(++))$. Clearly the quark doublets will not have zero modes and hence not acceptable. To complete the SM particle spectrum another T_1' of parity $(+-)$ to be introduced such

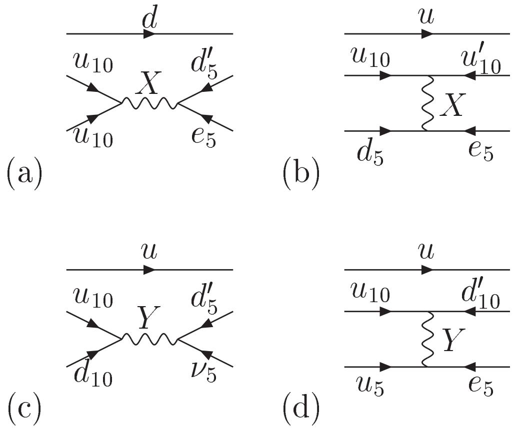

Figure 1: The proton decays through exchanging the X and Y gauge bosons. The subscripts denote SU(5) representations.

that the combination of the zero modes of (U_1^c, Q_1', E_1^c) and brane field F_1 constitutes the first SM family. In doing so the SU(5) mass relation will not be obeyed which is good phenomenologically. Henceforth we shall adopt the proposal that both T_1 and T_2 are bulk fields and no SU(5) mass relations holds for the first two generations.

Letting T_1 and T_2 go into bulk immediately implies a hierarchy between 4D effective mass of the third and the first two generations. The 5D Yukawa interaction can be written as

$$\mathcal{L}_5 \supset \frac{\tilde{f}_{uij}}{\sqrt{M^*/2}} \overline{\psi}^{\mathbf{c}}_{10i} \psi_{10j} H_5 + \frac{\tilde{f}_{dij}}{\sqrt{M^*/2}} \overline{\psi}^{\mathbf{c}}_{\bar{5}i} \psi_{10j} H_5^* + \text{H.c.} \tag{2.4}$$

with group indices suppressed and M^* denotes the fundamental scale. Because T_1 and T_2 live in the bulk, by naive dimension analysis, the 4D effective Yukawa coupling for down-type quarks naturally exhibit the following hierarchy patterns

$$y_d \sim c_d \begin{pmatrix} \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & 1 \\ \epsilon & \epsilon & 1 \end{pmatrix} \tag{2.5}$$

where the volume dilution factor $\epsilon \sim (M_c/M_*)^{1/2} \sim 0.1$ and c_d is a common factor. For the up-quarks, we adopt the following structure for 4D effective Yukawa couplings as proposed by [18]:

$$y_u \sim c_u \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} . \tag{2.6}$$

The volume dilution mechanism still works for the Yukawa interaction between one brane and one bulk fermions, i.e. $y_{u13}, y_{u23}, y_{u31}$ and y_{u32}. But one needs to assume the Yukawa couplings between T_1, T_2 to be two order of magnitude smaller then the others in order to have the structure of eq. (2.6). Given the totally different nature of T_1, T_2 from other fermions, this assumption is an ad hoc one and we have no deeper understanding why this has to be so.

The 5D bulk lagrangian is $Z_2 \times Z_2'$ symmetric. The question now arises: what parities should be assigned to the brane fermions? In our case the brane fermions will reside at $y = 0$, or equivalently $y = \pi R$, brane. They have definite parities under Z_2' transformation, such as

$$Z_2: \quad \Psi_{\bar{5}} \to +P\Psi_{\bar{5}}\,, \qquad \Psi_{10} \to +P^T\Psi_{10}P \tag{2.7}$$

$$Z_2': \quad \Psi_{\bar{5}} \to \pm P'\Psi_{\bar{5}}\,, \qquad \Psi_{10} \to \pm P'^T\Psi_{10}P' \tag{2.8}$$

So there are two ways of how the Z_2' parities can be assigned to brane fermions: (1) $(T, F) = (Q, U, E, D, L) = \pm(+--, -+)$ and (2) $(T, F) = (Q, U, E, D, L) = \pm(+--, +-)$. In order that the 5D lagrangian shall respect the symmetry of the theory we can simply make the follow manipulation on the brane term:

$$\int d^4x \int dy \frac{1}{2}\{\delta(y) \pm \delta(y - \pi R)\}L_\pm\,, \tag{2.9}$$

where subscript $+$ and $-$ stand for Z_2'-even and -odd respectively. Since our effective physical space is restrict to $y \in [0, \pi R/2]$, the choice of parity for the brane fermions do not make any difference.

Now we turn to the discussion of electroweak symmetry breaking. This is done by the bulk scalar in the **5** representation, H_5, to reduce G to $\mathrm{SU}(3) \times \mathrm{U}(1)_Q$ and to give charged fermion masses as seen in eq. (2.4). However, H_5 contains the color Higgs H_c with quantum number $(3, 1, -1/3)$ and the ordinary SM Higgs doublet H_w. The $(+-)$ parity of H_c forbids the existence of zero mode and hence no light color scalars on the y_S brane. Denoting $H_w = (h^+, h^0/\sqrt{2})^T$ as in SM, the Higgs doublet develops VEV $\langle H_w^T\rangle = (0, v_b^{3/2}/\sqrt{2})$ which breaks the electroweak symmetry as usual. Integrating out the fifth dimension, the effective gauge coupling can be used for relating the 5D and 4D gauge couplings, $g_4 = \tilde{g}/\sqrt{\pi R M^*}$. The resulting W boson mass is found to be $M_W^2 = g_2^2\pi R v_b^3/2$, or $(2\pi R v_b^3)^{1/2} = v_0 \sim 250\,\mathrm{GeV}$. With the above parameter substitution, we arrived at the effective 4D Yukawa interaction relevant to charged fermions masses:

$$\mathcal{L}_Y^4 = y_{u,ij}\frac{v_0}{\sqrt{2}}\bar{u}'_{Ri}u'_{Lj} + y_{d,ij}\frac{v_0}{\sqrt{2}}(\bar{d}'_{Ri}d'_{Lj} + \bar{e}'_{Rj}e'_{Li}) + \text{H.c.} \tag{2.10}$$

$$= \bar{u}'_R\mathcal{M}_u u'_L + \bar{d}'_R\mathcal{M}_d d'_L + \bar{e}'_R\mathcal{M}_e e'_L + \text{H.c.} \tag{2.11}$$

Where the mass matrices can be identified as:

$$\mathcal{M}_u = \frac{v_0}{\sqrt{2}}y_u\,, \qquad \mathcal{M}_d = \mathcal{M}_e^T = \frac{v_0}{\sqrt{2}}y_d\,. \tag{2.12}$$

The mass matrices are diagonalized by bi-unitary rotation:

$$U_R\mathcal{M}_u U_L^\dagger = \mathcal{M}_u^{dig}\,, \qquad V_R\mathcal{M}_d V_L^\dagger = \mathcal{M}_d^{dig}\,, \tag{2.13}$$

$$V_L^*\mathcal{M}_e V_R^T = \mathcal{M}_e^{dig}\,, \qquad V_{CKM} = U_L^\dagger V_L \tag{2.14}$$

and the leptons can be rotated into their mass eigenstates by

$$L_i = (V_R^*)_{ij}L'_j, \qquad e_{Ri} = (V_L^*)_{ij}e'_{Rj}\,. \tag{2.15}$$

Note that the neutrino is massless at this point, so we have the freedom to apply left-handed rotation to the entire lepton SU(2) doublet and make the charged current interaction diagonal. Also the eq. (2.5), (2.6) can not be taken literally otherwise the first two generations are degenerated. It shall be understood that every entity exhibits a small correction to the leading term exhibited. Numerical check shows that by allowing a 10% correction of each entry, it's easy to get the resulting mass hierarchy:

$$m_b : m_s : m_d = m_\tau : m_\mu : m_e \sim 1 : \epsilon : \epsilon^2 \,, \tag{2.16}$$

$$m_t : m_c : m_u \sim 1 : \epsilon^2 : \epsilon^4, (V_{us}, V_{cb}, V_{ub}) \sim (\epsilon, \epsilon, \epsilon^2) \,. \tag{2.17}$$

In our numerical experiments about one third of the statistical samples provide CKM matrix very close to experimental values. Hence, these two general mass matrix patterns successfully yield the observed charged fermions mass hierarchy and CKM mixing angles.

Since the lepton mixing is crucial in the mechanism for generating neutrino mass which to be discussed in section 3, more insight of the mixing matrix is required. It's easy to see that $V_L \sim diag(1,1,1)$, so the CKM is mainly controlled by the mixing of up-quark, U_L. The V_R is to diagonalize the almost uniform mass square matrix

$$\mathcal{M}_d \mathcal{M}_d^\dagger \sim \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{2.18}$$

For the leading approximation, the V_R is

$$V_R \sim \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \tag{2.19}$$

and it is equivalent to all its variants with any column permutation or arbitrary row sign flips because

$$\mathcal{M}_d \mathcal{M}_d^\dagger = \mathcal{S}_i \mathcal{M}_d \mathcal{M}_d^\dagger \mathcal{S}_i \,, \qquad (\mathcal{M}_d^{dig})^2 = \mathcal{G}_i (\mathcal{M}_d^{dig})^2 \mathcal{G}_i \tag{2.20}$$

where

$$\mathcal{S}_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} , \qquad \mathcal{S}_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \qquad \mathcal{S}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2.21}$$

correspond to the interchanging of V_R's 1-3, 1-2, and 2-3 columns. And $\mathcal{G}_1 = diag(-1,1,1)$, $\mathcal{G}_2 = diag(1,-1,1)$ and $\mathcal{G}_3 = diag(1,1,-1)$ are to flip sign of first, second and third row of V_R. This discrete symmetry is broken when one allows for small corrections to $\mathcal{M}_d$ as discussed before. Sometimes the first two indices switch to get the right order of charged lepton mass, $m_e < m_\mu$.

From our discussions above, we see there is a 5D non-SUSY orbifold SU(5) model setup which can give a good charged fermion mass hierarchy and mixing angles. Furthermore, this framework preserves the advantages of SU(5) GUT and avoid the major obstacles in the traditional 4D theories. Since it's not SUSY, the unification of coupling constants is nontrivial. We will defer the discussion of RG running and gauge unification to section 4. Now we turn to the neutrino masses problem.

3. Neutrino masses

It is well known that the symmetric **15** Higgs can be used to generate neutrino Majorana masses and break $(B-L)$ in conventional SU(5) theories [19]. For the orbifold version, we closely follow the study of [13] where a model of neutrino masses in 5D orbifold $\mathrm{SU}(3)_W$ unified theory without right handed singlet was constructed and the symmetrical **6** scalar filed was responsible for the neutrino masses. In the present case because $\mathbf{5}\times\mathbf{5}=\mathbf{10}+\mathbf{15}$, besides the **15** Higgs, the anti-symmetrical **10** Higgs can also work for generating the Majorana masses through one loop diagram, see figure 2.

Two things differ from the $\mathrm{SU}(3)_w$ case studied by [13]. Firstly, in the $\mathrm{SU}(3)_w$ theory both **3** and **6** contribute one Higgs doublet. One of the linear combination is the would be Goldstone boson to be eaten by the SM gauge bosons and the orthogonal linear combination is the required physical charged scalar that appears in the loop. In SU(5) case , the **10** and **15** break into

$$\mathbf{15} = \underbrace{\left(6,1,-\frac{2}{3}\right)}_{P_{15}} + \underbrace{\left(3,2,\frac{1}{6}\right)}_{C_{15}} + \underbrace{(1,3,1)}_{T} \tag{3.1}$$

and

$$\mathbf{10} = \underbrace{\left(\bar{3},1,-\frac{2}{3}\right)}_{P_{10}} + \underbrace{\left(3,2,\frac{1}{6}\right)}_{C_{10}} + \underbrace{(1,1,1)}_{S}\ . \tag{3.2}$$

in terms of SM quantum numbers. For notational simplicity, the symbols P, C, S, T have been introduced to indicate the specific component of **10** or **15** as shown in eqs. (3.1), (3.1). We see that neither **10** nor **15** contain a Higgs doublet component. So in the SU(5) case we have to introduce another $\mathbf{5}'$ Higgs for the radiative mechanism to work.

Secondly, due to the SU(5) symmetry which imposes strong constraints on model building the parity of exotic Higgs sector needs careful examination. Now we discuss the various possible parity assignments of **15** and **10**.

We first assign the **15** parity to be $(+-)$. The $\mathbf{15}(+-)$ will decompose into $P_{15}(+-) + C_{15}(++) + T(+-)$. By doing so, the triplet component T has no zero mode and hence no VEV can be developed. Hence, there is no tree level neutrino Majorana mass. Also, the hierarchy of the VEVs of SM Higgs doublet and the T Higgs triplet required by the ρ parameter measurement will be naturally avoided. In this case, the C_{15} color Higgs will improve the unification of non-SUSY SU(5) [20].

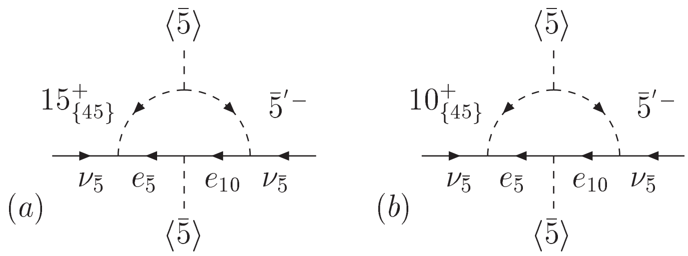

Figure 2: The 1-loop neutrino mass through $(a) H_5^{'\dagger} H_{15} H_5^*$ and $(b) H_5^{'\dagger} H_{10} H_5^*$ coupling.

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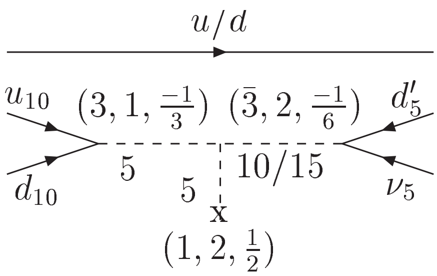

Figure 3: The proton and neutron decays through the mixing of $(3, 1, -1/3)$ color Higgs in $\mathbf{5}'$ and the $(3, 2, 1/6)$ color Higgs boson in **10** or **15**.

There are several shortcomings of this parity assignment. The Higgs boson running in the loop (see figure 2) are both KK modes due to KK number conservation. So the resulting neutrino masses are too small due to the suppression of KK mass $1/R$. Furthermore the parity of the extra $5'$ Higgs has to be $(+-)$ such that the bulk triple Higgs interaction

$$\frac{m}{\sqrt{M^*/2}} H_5^{'\dagger} H_{10/15} H_5^* + \text{H.c.} \tag{3.3}$$

is Z_2' invariant. Due to the above Higgs potential proton decay now will be induced via the mixing of the zero modes of C_{15} and the color component $(3, 1, -1/3)$ in $\mathbf{5}'$, see figure 3.

To satisfy both the constraints of proton decay and the requirement that the resulting 1-loop neutrino mass is of oder $.01\,\text{eV}$, the zero mode mass of C_{15} need to be as high as $10^{11}\,\text{GeV}$ and the triple Higgs coupling strength m is about $10^{16}\,\text{GeV}$. This scenario is disfavored by the strong interacting scalar sector and a very heavy scalar.

Next, the parity of **15** can be chosen as $(-+)$. Then the decomposition is $P_{15}(-+) + C_{15}(--) + T(-+)$. Again, the triplet T has no zero mode, so naturally avoid the VEV problem and the tree level neutrino mass. Now parity of the extra $5'$ Higgs has to be $(-+)$ correspondingly. But the lepton number breaking component vanish at $y = 0$ brane, so it doesn't couple to the $\bar{\mathbf{5}}$ living there. For this scenario to work, the $\bar{\mathbf{5}}$ fermions have to be placed at $y = \pi R/2$ and enjoy no SU(5) symmetry. Unfortunately, even doing so, the resulting neutrino mass pattern is not right.

The third parity assignment is $(--)$. This leaves the T components totally vanish at both branes. It does not work in our scenario where the F_i are localized on the brane.

So we are only left with the $(++)$ assignment. Explicitly, it satisfies the following transformation properties

$$P H_{15} P^{-1} = +H_{15}\,, \qquad P' H_{15} P^{'-1} = +H_{15}\,. \tag{3.4}$$

Now there are $P_{15}^{-2/3}(++)$, $(C_{15}^{+2/3}, C_{15}^{-1/3})(+-)$ and $(T^{+2}, T^{+1}, T^0)^T(++)$ Higgs. The T^0 couples two neutrinos which could give them the tree-level Majorana mass. To avoid that, we assume that it is a regular scalar and does not develop a VEV. Also, the zero mode of P_{15} must be heavy to suppress the $K - \bar{K}$ mixing, which can be gleamed from figure 4. This gives a satisfactory neutrino mass matrix and other phenomenological constraints.

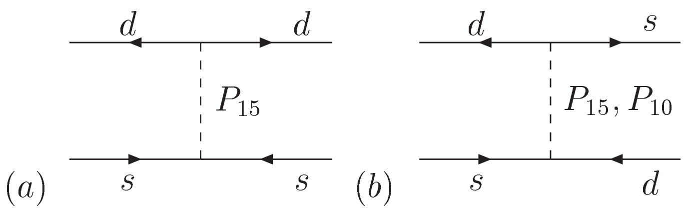

Figure 4: The tree level contribution to Kaon mass difference by exchanging the $(6, 1, -2/3)$ Higgs in **15** representation ((a),(b)) or the $(\bar{3}, 1, -2/3)$ Higgs in **10** representation ((b)).

Most of above discussions on the parity assignments can be applied to **10** so we will not repeat the analysis here but note that **10** doesn't acquire VEV. The suitable **10** is of $(++)$ parity under $Z_2 \times Z_2'$ too. It has components $P_{10}^{-2/3}(++)$, $(C_{10}^{+2/3}, C_{10}^{-1/3})(+-)$ and $S^{+1}(++)$.

Let's summarize the setup of the scalar sector:

1. One extra $\mathbf{5}'$ Higgs.
2. Either **15** or **10** scalar with parity $(++)$ under $(Z_2 \times Z_2')$.
3. The scalar **15** must not have VEV to avoid excessive fine tuning. The zero modes of scalar **15** or **10** must be heavy to avoid large contribution to $K - \bar{K}$ mixing.

The necessary lepton number violation terms arise from the brane Yukawa interactions

$$\mathcal{L}_{Y15} = \delta(y)\left[\frac{\tilde{f}^{15}_{ij}}{\sqrt{M^*/2}}\overline{\psi^{\{A\}c}_{\bar{5}i}}\psi^{\{B\}}_{\bar{5}j}H^{\{AB\}}_{15} + \text{H.c.}\right] \tag{3.5}$$

for **15** where $H^{\{AB\}}_{15} = H^{\{BA\}}_{15}$ and

$$\mathcal{L}_{Y10} = \delta(y)\left[\frac{\tilde{f}^{10}_{ij}}{\sqrt{M^*/2}}\overline{\psi^{\{A\}c}_{\bar{5}i}}\psi^{\{B\}}_{\bar{5}j}H^{\{AB\}}_{10} + \text{H.c.}\right] \tag{3.6}$$

for **10** and $H^{\{AB\}}_{10} = -H^{\{BA\}}_{10}$. After integrating out the extra dimension, we then parameterize the relevant 4D effective interaction as:

$$\mathcal{L} \supset \frac{\tilde{f}^{10}_{ij}}{\sqrt{\pi R M^*}}\overline{e'^c}_i\nu'_j S^{+1} + \frac{\tilde{f}^{15}_{ij}}{\sqrt{\pi R M^*}}\overline{e'^c}_i\nu'_j T^{+1} + \text{H.c.} \tag{3.7}$$

in the leptons' flavor basis. It is easy to see that $\tilde{f}^{15}_{ij} = \tilde{f}^{15}_{ji}$ and $\tilde{f}^{10}_{ij} = -\tilde{f}^{10}_{ji}$. The entries of the neutrino Majorana mass matrix can be approximately calculated by ignoring the lepton masses in the propagators and are given as follows:

$$(\mathcal{M})^{\nu}_{ij} = \frac{1}{16\pi^2}\frac{m(v_b)^{3/2}}{2\sqrt{\pi R M^*}\sqrt{M^*}}\sum_k \frac{m_k f'^5_{ik}\tilde{f}^{10/15}_{jk}}{M_1^2 - M_2^2}\ln\frac{M_2^2}{M_1^2} \tag{3.8}$$

where $\tilde{f}^{10/15}$ is the 5D Yukawa coupling of either **10** or **15**, m_k the mass of charged lepton-k and M_1, M_2 the masses of Higgs running in the loop. And f'^5 is the effective 4D Yukawa coupling for the relevant physical charged Higgs interaction:

$$f'^5_{ij}\bar{e}'_{Rj}\nu'_i H^- + \text{H.c.} \tag{3.9}$$

It is more convenient to express the neutrino mass matrix in the basis of charged lepton's mass eigenstates. Alone the line of the discussions in section2, we expect the lepton Yukawa coupling $(f'^5)^T$ to exhibit the same pattern as y_d in eq. (2.5). As we rotating the charged leptons into their mass eigenstate, the Yukawa matrix f'^5 is more or less diagonalized in the same time and it is proportional to the mass of charged lepton:

$$y'^5_{diag,ij} \sim \delta_{ij}\frac{g_2 m_i}{\sqrt{2}M_W}\,. \tag{3.10}$$

In the lepton mass eigenbasis, the Yukawa couplings for **10** or **15** can be obtained by the following transformations

$$f^{10} = V_R^\dagger\left\{\tilde{f}^{10}\right\}V_R^*\,, \qquad f^{15} = V_R^\dagger\left\{\tilde{f}^{15}\right\}V_R^*\,. \tag{3.11}$$

Only V_R is involved since we have invoked the SU(5) mass relation between the d^c and e. Note that the antisymmetry of flavor indices i, j of $\tilde{f}^{10}_{ij}$ and the symmetricalness of $\tilde{f}^{15}_{ij}$ is not affected by these transformations.

Therefore the eq. (3.8) is further simplified to

$$(\mathcal{M})^\nu_{ij} \sim \frac{g_2}{64\pi^2 M_W}\frac{m v_0}{(\pi R M^*)}\frac{m_i^2 f^{10/15}_{ij}}{M_1^2 - M_2^2}\ln\frac{M_2^2}{M_1^2} + (i \leftrightarrow j) \tag{3.12}$$

Thus, the matrix to diagonalize eq. (3.12) is the MNS matrix and we can directly use the phenomenological results of neutrino oscillation studies. With $v_0 \sim 250\,\text{GeV}$, $m_i \sim m_\tau$, $f^{10/15} \sim 1$, $M_1 \sim 10^5\,\text{GeV}$, $M_2 \sim 200\,\text{GeV}$ and $(\pi R M^*) \sim 100$, the overall neutrino mass can be estimated to be $0.13 \times (f^{10/15})(m/\text{TeV})\,\text{eV}$. It can easily give neutrino overall mass $\sim 0.01\,\text{eV}$ by adjusting $m \sim 0.1\,\text{TeV}$ such that the triple scalars is weakly interacting, see eq. (3.3).

For the case of **15**, the neutrino mass matrix takes the form:

$$\mathcal{M}^{15}_\nu \sim \frac{1}{M}\begin{pmatrix} f^{15}_{11}m_e^2 & f^{15}_{12}(m_e^2+m_\mu^2)/2 & f^{15}_{13}(m_e^2+m_\tau^2)/2 \\ f^{15}_{12}(m_e^2+m_\mu^2)/2 & f^{15}_{22}m_\mu^2 & f^{15}_{23}(m_\mu^2+m_\tau^2)/2 \\ f^{15}_{13}(m_e^2+m_\tau^2)/2 & f^{15}_{23}(m_\mu^2+m_\tau^2)/2 & f^{15}_{33}m_\tau^2 \end{pmatrix} \tag{3.13}$$

where M is a mass to be determined by eq. (3.12). Basically, it can yield any of the acceptable neutrino mass matrices. Here, we give few examples to illustrate the richness of resulting neutrino mass pattern by using **15**.

For instance, one solution is

$$\tilde{f}^{15} = \begin{pmatrix} 0.4959 & 0.2012 & -0.7524 \\ 0.2012 & 0.2717 & -0.4316 \\ -0.7524 & -0.4316 & 1.2360 \end{pmatrix} \tag{3.14}$$

which leads to a normal hierarchy mass pattern:

$$\overline{m}_0^{-1}\mathcal{M}_\nu = \begin{pmatrix} 3\times 10^{-5} & 0.4233 & -0.0271 \\ 0.4233 & 1.0 & 0.9319 \\ -0.0271 & 0.9319 & 1.1493 \end{pmatrix} \tag{3.15}$$

with $\theta_1 = 37.4°$, $\theta_2 = 44.04°$, $s_3 = -0.13$, $\triangle_\odot/\triangle_{atm} = 0.017$. $\overline{m}_0$ sets the overall neutrino mass scale which is determined by factors in eq. (3.12). In this example, $\overline{m}_0 = 0.05(m/0.15\text{TeV})\,\text{eV}$. Notice that the first entry will predict an unobservable rate for neutrinoless double beta decay. We have searched numerically for larger values of $(\mathcal{M}_\nu^{15})_{11}$ and found solutions with first entry as big as ~ 0.1. But they require fine tuning of Yukawa of order 10^{-4}. Even this value is below the sensitivity of the next generation of these experiments.

For neutrino mass matrices of the inverted hierarchy type, this mechanism for generating neutrino masses requires fine tuning of Yukawa couplings $\tilde{f}^{15}_{ij}$ which we will discuss below. Following the classification in [10, 5], we explore the following two leading patterns which lead to inverted hierarchy:

$$\mathcal{M}_\nu^{B1} \sim \frac{\overline{m}_0}{\sqrt{2}} \times \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \mathcal{M}_\nu^{B2} \sim \overline{m}_0 \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}. \tag{3.16}$$

Both of the leading patterns give bi-maximal mixing angles which conflict with the current experiments. Some small entries in the structure zeros as well as small perturbations to the leading terms are necessary to accommodate the experimental data.

Let's examine two examples: For $B1$ type, one solution for $\mathcal{M}_\nu$ is

$$\frac{\sqrt{2}}{\overline{m}_0}\mathcal{M}_\nu = \begin{pmatrix} 0.42 & 1 & 0.922 \\ 1 & 0.097 & -0.464 \\ 0.922 & -0.464 & 0.006 \end{pmatrix} \tag{3.17}$$

which gives $\theta_1 = 36.57°$, $\theta_2 = 42.43°$, $s_3 = 0.06$ and $\triangle_\odot/\triangle_{atm} = 0.021$ and implies

$$\tilde{f}^{15}_{B1} = \begin{pmatrix} 3\times 10^{-6} & 7\times 10^{-5} & -5\times 10^{-5} \\ 7\times 10^{-5} & 0.6667 & -0.4715 \\ -5\times 10^{-5} & -0.4715 & 0.3335 \end{pmatrix}. \tag{3.18}$$

One example of $B2$ type mass matrix is:

$$\frac{\mathcal{M}_\nu}{\overline{m}_0} = \begin{pmatrix} 1 & 0.02 & -0.01 \\ 0.02 & 0.49 & 0.5 \\ -0.01 & 0.5 & 0.5 \end{pmatrix} \tag{3.19}$$

which gives $\theta_1 = 34.36°$, $\theta_2 = 45.29°$, $s_3 = 0.021$ and $\triangle_\odot/\triangle_{atm} = 0.030$. The Yukawa coupling in the lepton weak basis is

$$\tilde{f}^{15}_{B2} = \begin{pmatrix} 6\times 10^{-6} & -3\times 10^{-6} & -6\times 10^{-6} \\ -3\times 10^{-6} & 0.6667 & -0.4714 \\ -6\times 10^{-6} & -0.4714 & 0.3333 \end{pmatrix}. \tag{3.20}$$

From the above examples we see that for both cases elements of the first row and first column are much smaller than the rest. This amounts to fine tuning of the model. We note in passing both inverted hierarchy types can be obtained from a Yukawa matrix with the leading structure that looks like

$$f^{15} \sim \begin{pmatrix} 1 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \tag{3.21}$$

where $\times$ denotes a small number of order 10^{-5}.

For **10**, the resulting neutrino mass matrix structure is:

$$\mathcal{M}_\nu^{10} \sim \frac{1}{M} \begin{pmatrix} 0 & f_{12}^{10}(m_\mu^2 - m_e^2)/2 & f_{13}^{10}(m_\tau^2 - m_e^2)/2 \\ f_{12}^{10}(m_\mu^2 - m_e^2)/2 & 0 & f_{23}^{10}(m_\tau^2 - m_\mu^2)/2 \\ f_{13}^{10}(m_\tau^2 - m_e^2)/2 & f_{23}^{10}(m_\tau^2 - m_\mu^2)/2 & 0 \end{pmatrix}. \tag{3.22}$$

Again, M is some mass to be determined by eq. (3.12). The diagonal zeros are the result of antisymmetry of the **10** representation. It can only give $B1$ type mass pattern, namely of the inverted hierarchy type, if the Yukawa exhibit the following hierarchy: $f_{d12}^{10} : f_{d13}^{10} : f_{d23}^{10} \sim 1 : m_\mu^2/m_\tau^2 : m_e^2/m_\tau^2$ which implies that

$$\tilde{f}^{10} \sim \begin{pmatrix} 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}. \tag{3.23}$$

The hierarchy is also a reflection of the approximate $L_\tau + L_\mu - L_e$ symmetry. It's impossible to get realistic neutrino masses by **10** alone at this leading order. However, small diagonal entries can be generated through 2-loop correction which we shall leave for future works. Another obvious mean is to include **15** as the perturbation source but this will lead to non-minimal models.

Hence, it is interesting that **15** gives a normal hierarchy whereas the **10** give inverted hierarchy of neutrino masses without additional symmetry or fine tuning of their respective Yukawa couplings.

4. Gauge unification

It is well known that the minimal 4D SU(5) unification is ruled out by current data. Even the supersymmetric version [21, 22] is disfavored [23]. With the extended Higgs sector required to generate neutrino masses we expect the unification of the gauge couplings will be a challenge. Since our model is a 5D one albeit non-supersymmetric, the renormalization group (RG) running of the SM gauge couplings crosses several thresholds and scales. We follow here the methods developed by the works of [25, 26]. Below the compactification scale, $1/R$, we have 4D effective field theory with thresholds crossings from the **15** or **10** mandated by our model of neutrino masses. This will constitute an intermediate scale between that of electroweak breaking and $1/R$. Between $1/R$ and the unification scale one has power law running of the 5D gauge theory. Clearly the problem of gauge unification now becomes highly nontrivial and requires careful treatment which we detail below. We will only discuss RG running and unification up to one loop.

The first task is to establish the lower limit on the mass of the **10** or the **15**, generically called M_P. We found the most stringent limit for M_P comes from the $K_L - K_S$ mass difference mediated by exchanging color Higgs components of $P_{10,15}$, see figure 4.

The $\triangle S \neq 0$ relevant terms are:

$$\frac{\tilde{f}^{10}_{ij}}{\sqrt{\pi R M^*}} \bar{d}^\alpha_i d^{\mathbf{c}\beta}_j P^{[\alpha\beta]}_{10} + \frac{\tilde{f}^{15}_{ij}}{\sqrt{\pi R M^*}} \bar{d}^\alpha_i d^{\mathbf{c}\beta}_j P^{\{\alpha\beta\}}_{15} + \text{H.c.} \tag{4.1}$$

where α, β are the color indices and it is understood the above expression shall be made SU(3) invariant (for details see [27]). The Yukawa interaction can be rotated into down quarks' mass eigenstates again by eq. (3.11) because the SU(5) symmetry relates left handed leptons and right handed down quarks. After applying the Fierz transformation, we arrive at the effective $\triangle S = 2$ operator from exchanging the P component of **15**:

$$\mathcal{L}^{\triangle S=2} \sim \frac{1}{2}\frac{1}{M_P^2}\left[\frac{f^{15}_{11} f^{15\dagger}_{22} + |f^{15}_{12}|^2}{\pi R M^*}\right] (\bar{d}^\alpha_R \gamma_\mu s^\alpha_R)(\bar{d}^\beta_R \gamma^\mu s^\beta_R) + \text{H.c.} \tag{4.2}$$

By using vacuum insertion approximation, the resulting kaon mass difference can be estimated to be:

$$\triangle M_K \sim \frac{2}{3} f_K^2 m_K Re\left(\frac{f^{15}_{11} f^{15\dagger}_{22} + |f^{15}_{12}|^2}{2M_P^2 \pi R M^*}\right). \tag{4.3}$$

The formula applies to **10** except that f^{10}_{ij} has no diagonal element. Plugging in the experimental values: $f_K \sim 0.16\,\text{GeV}$, $m_K = 0.4976\,\text{GeV}$ and $\triangle M_k = 3.48 \times 10^{-15}\,\text{GeV}$ we obtain the lower limit for M_P:

$$M_P > 1.11 \times 10^6 \left(\frac{f^{15}_{11} f^{15\dagger}_{22} + |f^{15}_{12}|^2}{\pi R M^*}\right)^{1/2} \left(\frac{\triangle M^P_K}{3.48 \times 10^{-15}\text{GeV}}\right)^{-1/2} \text{GeV}\,. \tag{4.4}$$

where $\triangle M^P_K$ is the mass difference due to color scalar contribution. Similar analysis can be naively extended to $\triangle M_B$ by substituting $f_K \leftrightarrow f_B$ and $M_K \leftrightarrow M_B$ which gives a less constrained limit:

$$M_P > 4.33 \times 10^5 \left(\frac{f^{15}_{11} f^{15\dagger}_{33} + |f^{15}_{13}|^2}{\pi R M^*}\right)^{1/2} \left(\frac{\triangle M^P_B}{3.75 \times 10^{-13}\text{GeV}}\right)^{-1/2} \text{GeV}\,. \tag{4.5}$$

Evidently, the limit goes down as the absolute value of specific Yukawa couplings of **15** or **10** is lower.

If we take the above number as the intermediate scale the question arises whether these color Higgs will induce rapid proton decay. It can be checked that there are neither tree level nor one loop contributions. Hence, with a relatively low value of $M_P \sim 10^5\,\text{GeV}$, obtained by assuming that $\pi R M^* \sim 100$, $\triangle M_K$ is saturated by the contribution of exotic scalars and the extreme case $|f^{10/15}| \sim 1$, will not run afoul of proton stability.

The one loop gauge coupling RG running after passing various thresholds can be written as

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{a_i^{SM}}{2\pi}\ln\frac{\mu}{M_Z} - \frac{a_i^H}{2\pi}\ln\frac{\mu}{M_H} - \frac{\tilde{a}_i^o}{2\pi}\sum_{n=1}^{N^o}\ln\frac{\mu R}{2n-1} - \frac{\tilde{a}_i^e}{2\pi}\sum_{n=1}^{N^e}\ln\frac{\mu R}{2n}\,. \quad (4.6)$$

The last two $\tilde{a}$ terms are the KK modes contributions when energy scale crosses $1/R$. The integers N^o and N^e are the highest odd ($(+-)$ and $(-+)$) and even ($(++)$ and $(--)$) KK excitation level below the scale μ

$$\frac{2N^e}{R} \le \mu\,, \qquad \frac{2N^o-1}{R} \le \mu\,. \quad (4.7)$$

For the SM, the beta functions are well known: $a_i^{SM} = (4, -10/3, -7) + n_H \times (1/10, 1/6, 0)$ where n_H is the number of Higgs doublet zero modes. At scale M_Z, deriving from $\alpha(M_Z)^{-1} = 127.934(7)$ and $\sin^2\theta_W = 0.231113(15)$ [28], we have

$$\alpha_1^{-1} = \frac{3}{5}\frac{\cos^2\theta_W}{\alpha} = 59.031(35)\,, \qquad \alpha_2^{-1} = \frac{\sin^2\theta_W}{\alpha} = 29.568(17) \quad (4.8)$$

and $\alpha_s^{-1} = 8.53$. In our model, the zero modes of **10** or **15** Higgs has a large mass M_P imposed by phenomenology. Below M_P, only SM particles go into the beta function. As one passes the threshold, $M_P < \mu < \frac{1}{R}$, the zero modes of **10** or **15** will contribute to the beta function. For **15**, it has zero modes $P_{15}(6, 1, -2/3)$ and $T(1, 3, 1)$ so

$$(a_1^{15}, a_2^{15}, a_3^{15}) = \left(\frac{17}{15}, \frac{2}{3}, \frac{5}{6}\right). \quad (4.9)$$

For **10**, the zero modes are $P_{10}(\bar{3}, 1, -2/3)$ and $S(1, 1, 1)$ hence

$$(a_1^{10}, a_2^{10}, a_3^{10}) = \left(\frac{7}{15}, 0, \frac{1}{6}\right). \quad (4.10)$$

Finally, when the scale is over the compactification scale, various KK excitations gradually come in and contribute to RG running. For very high KK levels, summing over the KK tower roughly gives the power law running of coupling constants. This power law behavior can be easily understood by adding up the KK excitations level by level. By simple algebra and Stirling's approximation, the KK sums are

$$\begin{aligned} S^o &= \sum_{n=1}^{N^o}\ln\frac{\mu R}{2n-1} = N^o\ln(\mu R) - \ln\frac{(2N^o-1)!}{(N^o-1)!} + (N^o-1)\ln 2 \\ &\sim N^o\ln(\mu R) - N^o\ln 2N^o + N^o - \ln\sqrt{2} + \mathcal{O}\left(\frac{1}{N^o}\right) \end{aligned} \quad (4.11)$$

$$\begin{aligned} S^e &= \sum_{n=1}^{N^e}\ln\frac{\mu R}{2n} = N^e\ln(\mu R) - \ln N^e! - N^e\ln 2 \\ &\sim N^e\ln(\mu R) - N^e\ln 2N^e + N^e - \ln\sqrt{2\pi N^e} + \mathcal{O}\left(\frac{1}{N^e}\right). \end{aligned} \quad (4.12)$$

Field	$\tilde{a}$	Even Components	$\tilde{a}^e_1$	$\tilde{a}^e_2$	$\tilde{a}^e_3$
H_5	1/6	$H_w(1,2,1/2)$	1/10	1/6	0
H_{15}	7/6	$P_{15}^{-2/3}(6,1,-2/3)$	8/15	0	5/6
		$T(1,3,1)$	3/5	2/3	0
H_{10}	1/2	$P_{10}^{-2/3}(\bar{3},1,-2/3)$	4/15	0	1/6
		$S(1,1,1)$	1/5	0	0
SU(5) Gauge	$-35/2$	$G^\mu(8,1,0)$	0	0	-11
		$W^\mu(1,3,0)$	0	$-22/3$	0
		$A^\mu(1,1,0)$	0	0	0
		$X^5,Y^5(3,2,-5/6)$	5/12	1/4	1/6
		$X^5,Y^5(\bar{3},2,5/6)$	5/12	1/4	1/6
T_{bulk}	2	$U^c(\bar{3},1,-2/3)$	16/15	0	2/3
		$E^c(1,1,1)$	4/5	0	0
T'_{bulk}	2	$Q'_L(3,2,1/6)$	2/15	2	4/3

Table 2: RG contribution from various KK excitations.

When $\mu \gg 1/R$, $N^o \sim N^e \sim \mu R/2$, the contribution of all the KK excitations to $\alpha_i^{-1}(\mu)$ can be approximately calculated as follows:

$$-\frac{\tilde{a}_i}{4\pi}(\mu R - \ln 2) + \frac{\tilde{a}^e_i}{4\pi}\ln\left(\frac{\pi\mu R}{2}\right) \tag{4.13}$$

where $\tilde{a}_i = \tilde{a}^e_i + \tilde{a}^o_i$ is the sum of the beta function from even and odd KK components. As μ being higher then the compactification scale the full SU(5) symmetry start to emerge. So it is not surprising that any complete SU(5) multiplet gives equal amount to three gauge running, $\tilde{a}_1 = \tilde{a}_2 = \tilde{a}_3$, which will not affect the gauge coupling unification. As unification is concerned, only the even (or equivalently the odd) KK components matter. Explicitly, we list the beta functions of all even KK contents in table 2.

Note that the fifth components of the KK X, Y gauge fields come in as real scalars which give one half of the contribution of complex scalars. Also note that the combined result of bulk filed T and T' are same to three coupling running so it will not affect unification. The contributions of KK modes can be summarized as:

$$(\tilde{a}^e_1, \tilde{a}^e_2, \tilde{a}^e_3) = \left(\frac{5}{6}, -\frac{41}{6}, -\frac{32}{3}\right) + n_5 \times \left(\frac{1}{10}, \frac{1}{6}, 0\right) + n_{15} \times \left(\frac{17}{15}, \frac{2}{3}, \frac{5}{6}\right) + n_{10} \times \left(\frac{7}{15}, 0, \frac{1}{6}\right) \tag{4.14}$$

and

$$\tilde{a} = -\frac{35}{2} + \frac{1}{6}n_5 + \frac{7}{6}n_{15} + \frac{1}{2}n_{10} + 4n_T \tag{4.15}$$

where n_5, n_{10}, n_{15} are the numbers of bulk $\mathbf{5}, \mathbf{10}, \mathbf{15}$ Higgs and n_T is the number of generation of ten-plet fermion in bulks respectively. With the minimum particle content we cannot obtain gauge unification as can be seen explicitly in figure 5.

To make this model work for unification, we propose to use vector fermions. The vector nature is to ensure no new anomaly is induced. They can be either n_Q pairs of heavy quark doublets, $Q_H + \overline{Q}_H$, coming from of $T_H(+-) + \overline{T}_H(+-)$ or n_L pairs of heavy

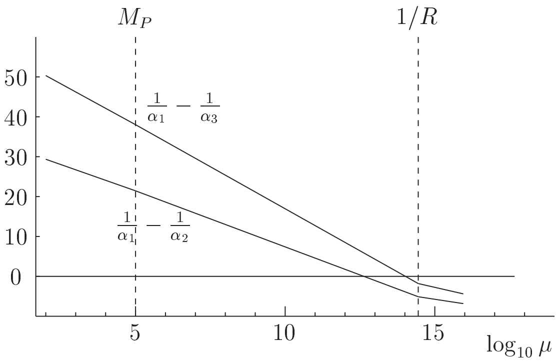

Figure 5: The RG running with $M_P = 10^5\,\mathrm{GeV}$ and $1/R = 2.81 \times 10^{14}\,\mathrm{GeV}$.

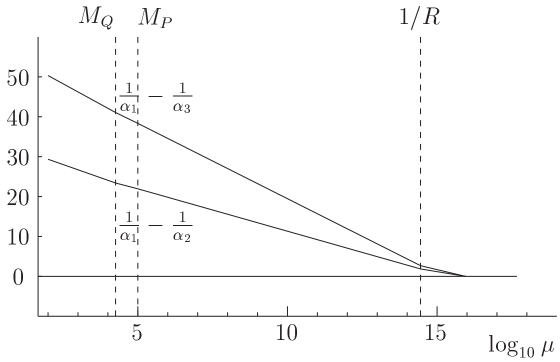

Figure 6: The RG running with $M_P = 10^5\,\mathrm{GeV}$, $M_Q = 18.03\,\mathrm{TeV}$ and $1/R = 2.81 \times 10^{14}\,\mathrm{GeV}$. The three couplings converge at $8.94 \times 10^{15}\mathrm{GeV}$.

lepton doublets $L_H + \overline{L}_H$ from bulk n_L pairs of $F_H(++) + \overline{F}_H(++)$. The new $F_H + \overline{F}_H$ must be engineered not to mix with the $F_{1,2,3}$, otherwise the neutrino mass matrix will be spoiled. For this reason we favored heavy quark pairs.

For simplicity, we assume they all have a common zero mode mass, denoted as M_{QL}, which can be adjusted to achieve unification. When $M_{QL} < \mu < 1/R$, the threshold effect of Q_Hs

$$(\triangle a_1, \triangle a_2, \triangle a_3) = 2n_Q \times \left(\frac{1}{15}, 1, \frac{2}{3}\right) \tag{4.16}$$

should be considered and the extra term

$$\triangle\alpha_i^{-1}(\mu) = -\frac{\triangle a_i}{2\pi} \ln \frac{\mu}{M_{QL}} \tag{4.17}$$

should be included in eq. (4.6). As the scale larger then $1/R$, their KK modes give extra contribution to $\tilde{a}^e$ and $\tilde{a}^o$ as

$$\triangle\tilde{a}_i^e = 2\triangle a_i\,, \qquad (\triangle\tilde{a}_1^o, \triangle\tilde{a}_2^o, \triangle\tilde{a}_3^o) = 4n_Q \times \left(\frac{14}{15}, 0, \frac{1}{3}\right) \tag{4.18}$$

where another factor 2 is due to that 5D fermions are vector like.

It turns out that a simple solution is with $n_Q = 1$. Now we have two **5** Higgs, one **15** Higgs with mass $M_P = 10^5\,\mathrm{GeV}$ to evade the constraint from $\triangle M_K$ and one pair of $Q_H + \overline{Q}_H$. If we require that $(\pi R M_{GUT}) = 100$, then the solution can be found to be $M_{QL} = 18.03\,\mathrm{TeV}$ and $M_{GUT} = 8.94 \times 10^{15}\mathrm{GeV}$, see figure 6.

Note the splitting of even and odd KK contribution slows down the convergence and push the unification scale to higher end. It is interesting to note that the volume factor, $\sim (\pi R M_{GUT})$, is related to M_{QL}. A higher M_{QL} provides smaller volume factor and lower M_{GUT}. For example, to have $(\pi R M_{GUT}) = 1000$ it requires $M_{QL} = 6.0\,\mathrm{TeV}$ and $M_{GUT} = 2.97 \times 10^{16}\,\mathrm{GeV}$. We have the approximate upper bound $M_{QL} < 117.1\,\mathrm{TeV}$ and $M_{GUT} > 1.16 \times 10^{15}\,\mathrm{GeV}$ from the consistency requirement that $\mu > 1/R$.

We note in passing that unification can still be made by adding many bulk **5** Higgs with parity (++) if heavy fermion doublets are not used. However, this solution requires a large number(≥ 6) of extra scalars needed. We deem this to be an unpalatable solution.

5. Phenomenology

The neutron anti-neutron oscillation may be induced by the mixing of Higgs, see figure 7. The relevant 6-quark operator can be expressed as

$$G_{\triangle B=2} u^c u^c d^c d^c d^c d^c + \text{H.c.} \tag{5.1}$$

the parameter $G_{\triangle B=2}$ is of mass dimension minus five. It can be estimated to be:

$$G_{\triangle B=2} \sim \frac{m f_5^2 f_{15}}{(1/R)^4 M_P^2 (\pi R M^*)} < \frac{10^{-52}}{(\pi R M^*)} \text{GeV}^{-5} \tag{5.2}$$

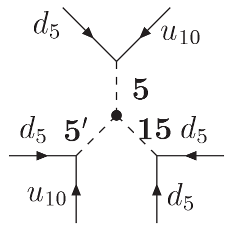

Figure 7: The Feynman diagrams for $N\bar{N}$ oscillation.

in arriving the result, the mass $M_P \sim 10^5$ GeV from the constraint of $\triangle M_K$, $f_5 \sim f_{15} \sim 1$, and $m \sim 1/R \sim 10^{14}$ GeV have been plugged in. It is safely within the experimental limit [30]

$$\tau_{N-\bar{N}} > 0.86 \times 10^8 sec \tag{5.3}$$

or equivalent, $G_{\triangle B=2} < 3 \times 10^{-28} \text{GeV}^{-5}$.

The lepton flavor violating muonium ($\mu^+ e^- \equiv M$)-antimuonium ($\mu^- e^+ \equiv \overline{M}$) transition and rare decay $\mu \to 3e$ can be induced at tree level by exchanging the $T^{\pm 2}$ scalars which belong to **15** Higgs, see figure 8. The interaction of T with charged leptons is given by

$$\mathcal{L}_T = \frac{f_{ij}^{15}}{\sqrt{\pi R M^*}} \bar{l^c}_i l_j T^{+2} + \text{H.c.} \tag{5.4}$$

where Yukawa coupling f^{15} is in the charged lepton's mass basis. Assuming there is no external electromagnetic fields and the mass difference of the mixed state, δ , is small. The possibility of observing a the transformation of muonium into antimuonium can be written as $P(\overline{M}) \sim \delta^2/2\Gamma_\mu^2$ where Γ_μ is the muon decay rate. And the mixing can be estimated to be

$$\delta \equiv 2\langle \overline{M} | \mathcal{H}_{M\overline{M}} | M \rangle = \frac{2 f_{11}^{15} f_{22}^{15\dagger}}{\pi a^3 M_T^2 (\pi R M^*)} \tag{5.5}$$

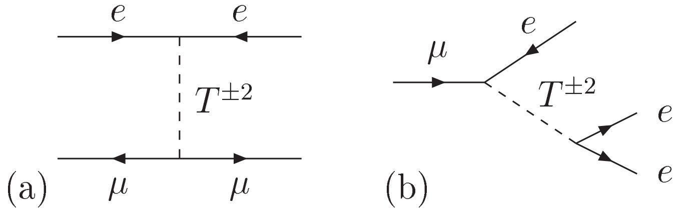

Figure 8: The Feynman diagrams for (a) muonium-antimuonium and (b) $\mu \to 3e$ transition by exchange Higgs $T^{\pm 2}$.

where a is the Bohr radius ($a^{-1} = \alpha m_e$). $P(\overline{M})$ can be expressed as

$$P(\overline{M}) \sim 1.1 \times 10^{-16} \frac{|f^{15}_{11} f^{15\dagger}_{22}|^2}{(\pi R M^*)^2} \left(\frac{M_T}{10^5 \mathrm{GeV}} \right)^{-4} \tag{5.6}$$

which is safely within the current experimental limit $P(\overline{M}) < 8.3 \times 10^{-11}$ [29].

The $\mu \to 3e$ transitions can be described by an effective lagrangian

$$\frac{(f^{15\dagger}_{11} f^{15}_{12})}{M_T^2 (\pi R M^*)} (\bar{e^c} \mu_L)(\bar{e} e^c) + \mathrm{H.c.} \tag{5.7}$$

which leads to the branching ratio of $\mu \to 3e$:

$$Br(\mu \to 3e) = \frac{2|f^{15\dagger}_{11} f^{15}_{12}|^2}{g_2^4 (\pi R M^*)^2} \left(\frac{M_W}{M_T} \right)^4 . \tag{5.8}$$

The above equation can be expressed in term of $\triangle M_K$ to eliminate the uncertainty of the absolute value of f^{15}:

$$Br(\mu \to 3e) = 2 \left(\frac{3 M_W^2 \triangle m_K}{g_2^2 f_K^2 m_K} \right)^2 \times \left(\frac{\triangle m_K^P}{\triangle m_K} \right)^2 \times \left(\frac{M_P}{M_T} \right)^4 \times \left| \frac{|f^{15\dagger}_{11} f^{15}_{12}|}{f^{15\dagger}_{11} f^{15}_{22} + |f^{15}_{12}|^2} \right|^2 \tag{5.9}$$

where $\triangle m_K^P$ is the contribution to kaon mass difference by exchanging P scalar. We give explicit dependence of the ratio of M_P to M_T because we expect they will split after quantum corrections are taken into account. In the above we have used the relations between Yukawa couplings and the elements of the neutrino mass matrix, see eq. (3.13). So the ratio of Yukawa couplings can be replaced by the ratio of the corresponding elements in $\mathcal{M}_\nu$:

$$Br(\mu \to 3e) \sim 3.02 \times 10^{-16} \left(\frac{\triangle m_K^P}{\triangle m_K} \right)^2 \left(\frac{M_P}{M_T} \right)^4 \left(\frac{2 m_{11} m_{12}}{m_{11} m_{22} + (2 \frac{m_e}{m_\mu} m_{12})^2} \right)^2 . \tag{5.10}$$

It's straightforward to extend the analysis to $\tau \to 3l$ transitions. Assuming that the hierarchy of the elements of neutrino mass matrix is smaller than factor 100, this model predicts

$$\begin{aligned} &Br(\mu \to 3e) : Br(\tau \to 3e) : Br(\tau \to 3\mu) : Br(\tau \to \mu e e) : Br(\tau \to e \mu \mu) \\ &\qquad \sim \frac{m_{12}^2}{m_{22}^2} : \left(\frac{m_\mu}{m_\tau} \right)^4 \frac{m_{13}^2}{m_{22}^2} : \left(\frac{m_e}{m_\tau} \right)^4 \frac{m_{23}^2}{m_{11}^2} : \left(\frac{m_\mu}{m_\tau} \right)^4 \frac{m_{23}^2}{m_{22}^2} : \left(\frac{m_e}{m_\mu} \right)^4 \frac{m_{12}^2}{m_{11} m_{22}} . \end{aligned} \tag{5.11}$$

The extra suppression of mass ratio to the fourth power makes it very difficult to find experimental signal in $\tau \to 3l$ decays. Hence, $\mu \to 3e$ is the best probe of the model. But this model exhibits an interesting characteristic: only the neutrino mass matrices of $B1$ type have the chance to be benefited from large enhancement $(m_{12}/m_{22})^2$ such that $\mu \to 3e$ can be observed in near future experiments.

The rare decays $\mu \to e\gamma$, $\tau \to \mu\gamma$, $b \to s\gamma$ etc can be induced by one loop diagrams which involve corresponding components of either **15** or **10**. Clearly, these rare decays are useful tools for probing flavor physics. But, again, these process are suppressed by $(M_W/M_P)^4$ plus the loop factor suppression which make their rates too small to be tested in foreseeable experiments.

What about seeing the effects of the heavy quarks $Q_H + \overline{Q}_H$? Since we expect their masses to be heavier then 10 TeV direct production of these particles is not likely at the LHC. One can look for their virual effects. Because they are vector like so they have no leading order contribution to the S and T parameters [31].

6. Conclusions

We have explicitly constructed viable models of neutrino mass matrix involving only three active SM Weyl neutrinos without introduction of singlet right-handed fermion in 5D orbifold SU(5) unification models. The crucial source of lepton number violation is the **15** or **10** bulk Higgs fields. This model preserves the orbifold solution to the triplet-doublet problem and avoids rapid proton decay mediated by leptoquarks at the tree level due to KK number conservation at the vertex coupling two **10** fermions which are bulk fields. However, at the one loop level proton decay can occur. Our estimate puts it safely within the experimental bound.

In the class of models we studied the overall scale of neutrino mass is partly controlled by the Yukawa couplings of the exotic Higgs bosons to the brane leptons and triple scalar coupling and the mass scale of the exotic Higgs fields. It is of order 10^{-2} eV after all the phenomenological constraints are satisfied. We found that the **15** prefers a normal hierarchy; whereas the **10** gives an inverted hierarchy when two loop effects are taken into account. Otherwise only bi-maximal mixing can be obtained. Since the model is restricted to three Weyl neutrinos the mass matrix obtained is necessary Majorana. This is a necessary but not sufficient condition to induce neutrinoless double beta decay of nuclei. To do so the (11) entry of the mass matrix must be non vanishing. Generically without fine tuning of the Yukawa couplings, it is typically of order 10^{-4} eV which puts it outside the range of detectability in the next round of experiments. This is a reflection of the fact that normal hierarchy or Zee-like mass matrices are preferred. Stating this differently, if a value of $|m_{ee}| \sim 0.1$ eV is extracted from the next generation of experiment which we imagine to give positive signatures then we would require a 4 orders of magnitude hierarchy in the Yukawa couplings of **15** for this mechanism to work.

As expected the introduction of exotic Higgs exacerbates the problem of gauge unification of SU(5). We found a solution to this in the 5D model by using a pair of vector bulk fermions **10** and $\overline{\mathbf{10}}$. The masses of these fermions are in the 10 TeV range and unification occurs at $\sim 10^{16}$ GeV and the compactification radius $1/R \sim 10^{14}$ GeV. On the other hand, these exotic scalars are promising source to give the universe enough matter anti-matter asymmetry. Detailed analysis of this will appear elsewhere [32].

In our study we have not considered in detail the origin of the charged fermion mass hierarchy. It is sufficient for us to put the third family on the SU(5) brane and the first two families **10** fermions as bulk fields. Perhaps this can solved by the split fermion scenario similar to [27] but this we leave for future considerations.

It is well known challenge to test the physics of various models for neutrino masses. Currently the data do not distinguish the different classes of models let alone the many within one framework. For the seesaw model, generally the right-handed Majorana neutrinos are of order of GUT scale and direct probe is out of the question. For the case of bulk right-handed neutrinos studies thus far done have indicated phenomenological tests are only possible if the compactification is very large [6, 33]. Since the bulk neutrino behaves like a sterile ones more structures in the oscillation pattern will be a good signature [34]. Currently no such structure is seen and more precise measurements will be needed. In our radiative scenario it is possible to see the effects of lepton number violating Higgs in rare decay. We found that once the mass M_P satisfies the constraint from $\triangle M_K$ then most of the lepton and baryon flavor violating transition induced by **15** are far below the present experimental limits. This also puts it out of the range of direct detection in the near future. However, the decay $\mu \to 3e$ can be just below the current experimental limit. If found may indicate the neutrino mass matrix is of $B1$ type inverted hierarchy which leads to a very low neutrinoless double beta decay rate.

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